Name: RED ID:

Spring 2024 Math 245 Exam 3

Please read the following directions:

Please write legibly, with plenty of white space. Please **print** your name and REDID in the designated spaces above. Please fit your answers into the designated areas; material outside the designated areas (such as on this cover page) will not be graded. To get credit, you must also show adequate work to justify your answers. If unsure, show the work. All problems are worth 6-12 points, for a minimum score of 60/120 and a maximum score of 120/120. The use of notes, books, calculators, or other materials on this exam is strictly prohibited, except you may bring one 3"x5" card (both sides) with your handwritten notes. This exam will begin at 10:00 and will end at 10:50; pace yourself accordingly. Good luck!

Special exam instructions for SSW-1500:
1. Please don't leave empty seats in the middle of the classroom – they will only get filled after the exam starts, which is better for nobody. As usual, please leave the seats on the far right for latecomers.
2. Please stow all bags/backpacks/boards at the front of the room. All contraband, except phones, must be stowed in your bag. All smartwatches and phones must be silent, non-vibrating, and either in your pocket or stowed in your bag.
3. Please remain quiet to ensure a good test environment for others.
4. Please keep your exam on your desk; do not lift it up for a better look.
5. If you have a question or need to use the restroom, please come to the front. Bring your exam. I cannot come to you unless you are sitting by an aisle, sorry.
6. If you are done and want to submit your exam and leave, please wait until one of the three designated exit times, listed below. Please do **NOT** leave at any other time. If you are sure you are done, just sit and wait until the next exit time, with this cover sheet visible.

Designated exam exit times:
- 10:20 "I need to work harder"
- 10:40 "I can't wait to get out of here"
- 10:50 "I need every second I can get"

REMINDER: Use complete sentences.

Problem 1. Carefully define the following terms:

a. symmetric difference

b. union

Problem 2. Carefully define the following terms:

a. disjoint

b. antisymmetric

Problem 3. Find a partition of $\mathbb{N}$ into infinitely many parts, each of a different cardinality (from each other). You need not prove it is a partition, just find it.

Problem 4. Let $R = \{x \in \mathbb{Z} : \exists y \in \mathbb{Z}, x = 2y\}, S = \{x \in \mathbb{Z} : \exists y \in \mathbb{Z}, x = 6y\}, T = \{x \in \mathbb{Z} : \exists y \in \mathbb{Z}, x = 3y\}$. Prove or disprove that $R \cap S = T$.

Problem 5. Let S, T be sets, and suppose that $S \setminus T = T \setminus S$. Prove that $S \subseteq T$.

Problem 6. Let A, B be nonempty sets, and suppose that $A \times B \subseteq B \times A$. Prove that $A = B$.

Problem 7. Let S, U be sets with $S \subseteq U$. Prove that $S \subseteq (S^c)^c$.
NOTE: This is part of Theorem 9.2. Do not use this theorem to prove itself!

For problems 8-10, let $S = \{a\}$, $V = 2^S$, and $W = 2^V$. Define relation R on W via $R = \{(x, y) : x \subseteq y\}$. Each of these problems has two parts.

Problem 8. Draw the digraph for relation R. Also, determine $|R|$.

Problem 9. Prove or disprove that R is reflexive. Also, prove or disprove that R is symmetric.

Problem 10. Let R_{symm} denote the symmetric closure of R. Draw the digraph for relation R_{symm}. Also, prove or disprove that R_{symm} is transitive.